Quantum Field Theory	**Exercises No. 13**

[Wednesday 13. July 2010, 12:30-14:00 in D5-153]

Exercise 13.1: Consider a complex scalar field theory in Euclidean space-time

$$\mathcal{L}_E = \partial_\mu \phi^* \partial_\mu \phi - V(\phi^* \phi) \ .$$

The theory is invariant under the global transformation $\phi \to e^{i\omega}\phi$, $\phi^* \to e^{-i\omega}\phi^*$.

Derive the Noether current by

(a) using the relation from lecture $\frac{\delta S_E}{\delta \omega(x)} \equiv -\partial_\mu j_\mu(x)$.

(b) by comparison with ex. 1.4

Exercise 13.2: Given are the linearly realised symmetries

$$\delta \phi^a(x) = \Phi_i^a(x) \delta \omega^i \equiv T_i^{ab} \phi^b(x) \delta \omega^i \ ,$$

with constant coefficients T_i^{ab}, such that $\partial S_E[\phi^a]/\partial \omega^i = 0$ holds. Show that the following relations hold:

(a) $\int d^4x\, T_i^{ab} \phi^b(x) \frac{\delta S_E[\phi]}{\delta \phi^a(x)} = 0$,

(b) $\int d^4x\, T_i^{ab} J^a(x) \frac{\delta Z[J]}{\delta J^b(x)} = 0$,

(c) $\int d^4x\, T_i^{ab} \varphi^b(x) \frac{\delta \Gamma[\varphi]}{\delta \varphi^a(x)} = 0$.

[Thus effective action $\Gamma[\varphi]$ and classical action $S_E[\phi]$ share the same symmetries.]

Exercise 13.3: Use the Jacobi identity to show that the following relation holds for the BRST transformations given in lecture:

$$s\Big(\mathcal{D}_\mu^{ab} c^b\Big) = 0$$

Exercise 13.4: Show that

$$s\Big(\frac{g}{2} f^{abc} c^b c^c\Big) = 0 \ ,$$

with c being Grassmann variables and the trafo fromlecture. This implies the nil-potency of the transformation $s^2 c^a = 0$.