

Exercise 1 - Andreief formula

Let $\varphi_j(x)$ and $\psi_j(x)$ with $j = 1, \dots, N$ be two sets of integrable functions. Then the Andreief formula states

$$\prod_{k=1}^N \int dx_k \det_{1 \leq i, j \leq N} [\varphi_i(x_j)] \det_{1 \leq i, j \leq N} [\psi_i(x_j)] = N! \det_{1 \leq i, j \leq N} \left[\int dx \varphi_i(x) \psi_j(x) \right] \quad (1)$$

Prove this formula.

Exercise 2 - An algebraic identity

Prove:

$$(y - x)^4 = \det \begin{pmatrix} 1 & 0 & 1 & 0 \\ x & 1 & y & 1 \\ x^2 & 2x & y^2 & 2y \\ x^3 & 3x^2 & y^3 & 3y^2 \end{pmatrix}. \quad (2)$$
