Introduction to Random Matrices	**Exercise Sheet No. 10**	07.01.2019

Exercise 1 - Expansion of the first and second eigenvalue distribution

Write out the first few terms of the expansion of the 1st and 2nd eigenvalue distribution $p^{(\beta)}_{1,2}(s)$. Note: The expansion terms result from the expansion of the corresponding gap probabilities.

Exercise 2 - Saddle point equation for the resolvent and the semi-circle

Complete the derivation of the saddle point equation for the resolvent

$$G_N(z) = \frac{1}{N}\mathrm{Tr}\,\frac{1}{z-H} \tag{1}$$

from the lecture. The saddle point equation was given as

$$zG_N(z) - 1 = \frac{1}{2}G_N^2(z) + \frac{1}{2N}G'_N(z) \tag{2}$$

Show why the above equation has two solutions in the large-N limit

$$\lim_{N\to\infty}\langle G_N(z)\rangle = G_\infty^{(\mathsf{av})}(z) = z \pm \sqrt{z^2-2} \tag{3}$$

and explain why only the minus-solution gives the limiting resolvent.
Use the inversion formula from the lecture, i.e.

$$\rho(z) = \frac{1}{\pi}\lim_{\varepsilon\to 0^+}\mathrm{Im}\,G_\infty^{(\mathsf{av})}(z-i\varepsilon) \tag{4}$$

to derive the semi-circle $\rho_{SC}(x) = \frac{1}{\pi}\sqrt{2-x^2}$ from the limiting resolvent.

Exercise 3 - Loop equation derivation

Determine the behaviour of $\mathrm{Tr}\,v(H)$, where $v(H)$ is supposed to be an analytic potential under the change of variables

$$H \to H + \varepsilon\frac{1}{z-H} \tag{5}$$

to leading order in ε.
Do the same for the integration measure $[dH]$.
