Exercise 1 - Planar solution for the resolvent

Recall the formula for the zero (planar) solution of the resolvent $G_0(z)$ for arbitrary potential $v(z)$, i.e.

$$G_0(z) = \frac{1}{2}\left(v'(z) - M(z)\sqrt{(z-a)(z-b)}\right) \tag{1}$$

with support $\sigma = [b, a]$ and $M(z)$ given by

$$M(z) = \oint_{C_\infty} \frac{dw}{2\pi i} \frac{v'(z)}{w-z} \frac{1}{\sqrt{(w-a)(w-b)}} \tag{2}$$

- Recover the semi-circle $\rho_{SC}(x) = \frac{1}{\pi}\sqrt{2-x^2}$ from $G_0(z)$ with the potential

  $$v(x) = \frac{1}{2} g_2 x^2$$

  and the condition $a = -b$ (for suitable a).

- Try to determine $G_0(z)$ for the potential

  $$v(x) = \frac{1}{2} g_2 x^2 + \frac{1}{4} g_4 x^4$$

  for suitable support $\sigma = [b, a]$.
