Exercise 1 - Laguerre polynomials

Derive the 1-point correlation function via

$$R_{N,1}(x) = x^{\alpha} e^{-x} K_N^{\alpha}(x,x) \tag{1}$$

where the corresponding orthogonal polynomials are the classical Laguerre polynomials $L_n^{\alpha}(x)$ with their corresponding differential equation. Furthermore show that the gap probability $E_0(s)$ for $\alpha = 1$ in this case is given by

$$E_0^{\alpha=1}(s) = e^{-Ns} L_N^{\alpha=0}(-s) \tag{2}$$

Exercise 2 - Large N-limit at the origin and Bessel kernel

Recall the quarter circle picture from the lecture. Show that the distribution of the first eigenvalue has the following large N-asymptotics

$$p_1^{\alpha=1}(y) = \frac{y}{2} I_2(y) e^{-\frac{y^2}{4}} \tag{3}$$

where $I_{\nu}(y)$ denotes the modified Bessel function of the first kind.
Show also that the limit kernel (called the Bessel kernel) has the form

$$K_s(y_1, y_2) = \frac{J_{\alpha}(y_1) y_2 J_{\alpha-1}(y_2) - J_{\alpha}(y_2) y_1 J_{\alpha-1}(y_1)}{y_1^2 - y_2^2} \tag{4}$$
