

[ Tutor: Marc Sangel, Tutorial: Wednesday 20. April 2016, 16:00-17:30 in V2-200 ]

**Exercise 1.1:** Consider a generic Lagrange density  $\mathcal{L}(\phi, \partial_\mu \phi)$  that is not explicitly  $x$ -dependent.

(a) Verify that  $\mathcal{L}$  is invariant under the substitution

$$x^\mu \rightarrow x^{\mu'} = x^\mu + \omega^\mu, \quad \phi(x) \rightarrow \phi'(x') = \phi(x), \quad (1)$$

where  $\omega^\mu$  is  $x$ -independent.

(b) What are the corresponding Noether currents  $j_\nu^\mu$ ?

(c) In this case we denote by  $j_\nu^\mu \equiv -T^\mu{}_\nu$ . Show that  $T^{00} = \mathcal{H}$ .

**Exercise 1.2:** Let  $\partial_\mu j_i^\mu = 0$ . Under which conditions is the "charge"  $Q_i \equiv \int_V d^3\vec{x} j_i^0$  a conserved quantity?

**Exercise 1.3:** Consider the Lorentz transformation  $x^\mu \rightarrow x^{\mu'} \equiv \Lambda^\mu{}_\nu x^\nu$ , where  $\eta^{\alpha\beta} \Lambda^\mu{}_\alpha \Lambda^\nu{}_\beta = \eta^{\mu\nu}$ . We assume that field  $\phi$  remains invariant:  $\phi(x) \rightarrow \phi'(x') \equiv \phi(x)$ .

(a) Show that  $S = \int d^4x \mathcal{L}$  with  $\mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - V(\phi)$  remains invariant under this Lorentz transformation.

(b) Demonstrate that an infinitesimal  $\Lambda^\mu{}_\alpha$  can be written in the form  $\Lambda^\mu{}_\alpha = \delta^\mu{}_\alpha + \epsilon^\mu{}_\alpha$ , where  $\epsilon^{\mu\nu} = -\epsilon^{\nu\mu}$ .

(c) What are the corresponding Noether currents?

**Exercise 1.4:** Let us consider the following Lagrange density for the two fields  $\phi_{1,2}$

$$\mathcal{L} = \frac{1}{2} \partial^\mu \phi_a \partial_\mu \phi_a - V(\phi_a^2), \quad (2)$$

(summation convention!) and the substitution

$$\delta\phi_1 \equiv -\phi_2 \delta\omega, \quad \delta\phi_2 \equiv \phi_1 \delta\omega. \quad (3)$$

Verify that this is a symmetry of the Lagrange density (2). What is the Noether current?