

[Tutor: Marc Sangel, Tutorial: Wednesday 04. May 2016, 16:00-17:30 in V2-200]

Exercise 3.1: Show that the following holds:

$$(a) \quad \int_{-\infty}^{\infty} \frac{d\tilde{k}}{2\pi} \frac{e^{i\tilde{k}\tau}}{\tilde{k}^2 + E^2} = \frac{1}{2E} \left[\theta(-\tau)e^{E\tau} + \theta(\tau)e^{-E\tau} \right],$$

$$(b) \quad \lim_{\epsilon \rightarrow 0^+} \int_{-\infty}^{\infty} \frac{dk}{2\pi} \frac{e^{ikt}}{k^2 - E^2 + i\epsilon} = \frac{-i}{2E} \left[\theta(-t)e^{iEt} + \theta(t)e^{-iEt} \right].$$

Hint: use Cauchy's integral formula.

Exercise 3.2: The Fourier transformation of the Schwinger propagator is given by

$$\tilde{G}_E(\tilde{p}^0; \vec{p}) = \frac{1}{(\tilde{p}^0)^2 + E_{\vec{p}}^2},$$

and the spectral function is defined as

$$\tilde{\rho}(p^0; \vec{p}) = \frac{\pi}{2E_{\vec{p}}} \left[\delta(p^0 - E_{\vec{p}}) - \delta(p^0 + E_{\vec{p}}) \right].$$

Demonstrate that the following relation holds:

$$\tilde{\rho}(p^0, \vec{p}) = \Im m[\tilde{G}_E(\tilde{p}^0 \rightarrow -i(p^0 + i0^+); \vec{p})].$$

Hint: recall different definitions of the delta-function.

Exercise 3.3: Let $\hat{U}_I(t, t_0)$ be the time evolution operator as defined in lecture.

- Using its definition express $\hat{U}_I(t, t_0)$ in terms of the following operators $e^{i\hat{H}t}$, $e^{i\hat{H}t_0}$, $e^{i\hat{H}_0t}$, $e^{i\hat{H}_0t_0}$.
- Demonstrate that for any times t_1, t_2 it holds $\hat{U}_I^\dagger(t_1, t_2) = \hat{U}_I(t_2, t_1)$.
- Show that $\hat{U}_I(t_1, t_2)\hat{U}_I(t_2, t_3) = \hat{U}_I(t_1, t_3)$ holds for $t_1 \geq t_2 \geq t_3$.
- Show that $\hat{U}_I(t_1, t_3)\hat{U}_I^\dagger(t_2, t_3) = \hat{U}_I(t_1, t_2)$ holds for $t_1 \geq t_2 \geq t_3$.
- Using these results show that the scattering matrix \hat{S} defined in lecture is unitary

$$\hat{S}^\dagger \hat{S} = \hat{S} \hat{S}^\dagger = \mathbf{1}.$$