Quantum Field Theory	**Exercises No. 5**	12.05.2016

[Tutor: Marc Sangel, Tutorial: Wednesday 18. May 2016, 16:00-17:30 in V2-200]

Exercise 5.1: Check Wick's Theorem to third order:

$$\begin{aligned} T\{\hat{\phi}_I(x_1)\hat{\phi}_I(x_2)\hat{\phi}_I(x_3)\} &= \,:\hat{\phi}_I(x_1)\hat{\phi}_I(x_2)\hat{\phi}_I(x_3): \\ &\quad + :\hat{\phi}_I(x_1): \langle 0|T\{\hat{\phi}_I(x_2)\hat{\phi}_I(x_3)\}|0\rangle \\ &\quad + :\hat{\phi}_I(x_2): \langle 0|T\{\hat{\phi}_I(x_1)\hat{\phi}_I(x_3)\}|0\rangle \\ &\quad + :\hat{\phi}_I(x_3): \langle 0|T\{\hat{\phi}_I(x_1)\hat{\phi}_I(x_2)\}|0\rangle \ . \end{aligned}$$

Exercise 5.2: Let $\mathcal{L}_{\text{int}} = -\frac{1}{3!}\lambda\phi^3$. Determine $G^{(3)}_{T,C}(x_1, x_2, x_3)$ up to including order $\mathcal{O}(\lambda^1)$ by counting all diagrams as in lecture. Which diagrams are connected? Check that you counted all contractions.

Exercise 5.3: Let

$$\begin{aligned} G^{(n)}_E(\tau_1, ..., \tau_n) &\equiv \langle 0|T\{\hat{x}_H(\tau_1)\dots\hat{x}_H(\tau_n)\}|0\rangle \ , \\ G^{(n)}_\beta(\tau_1, ..., \tau_n) &\equiv \frac{\mathrm{Tr}\,[e^{-\beta\hat{H}}T\{\hat{x}_H(\tau_1)\dots\hat{x}_H(\tau_n)\}]}{\mathrm{Tr}\,[e^{-\beta\hat{H}}]} \ , \end{aligned}$$

with $0 \le \tau_1, ..., \tau_n \le \beta$. Show that it holds: $\lim_{\beta\to\infty} G^{(n)}_\beta(\tau_1, ..., \tau_n) = G^{(n)}_E(\tau_1, ..., \tau_n)$.

Exercise 5.4: Starting from

$$e^{W(J)} \equiv \int \mathrm{d}\vec{v}\, \exp\Big[-\frac{1}{2}v_i A_{ij} v_j + J_i v_i\Big] = e^{W(0)} \exp\Big[\frac{1}{2}J_i A^{-1}_{ij} J_j\Big] \ ,$$

compute

$$\langle v_m v_n v_o v_p\rangle_0 \equiv \frac{\int \mathrm{d}\vec{v}\ v_m v_n v_o v_p\ \exp\Big[-\frac{1}{2}v_i A_{ij} v_j\Big]}{\int \mathrm{d}\vec{v}\ \exp\Big[-\frac{1}{2}v_i A_{ij} v_j\Big]} \ .$$