Quantum Field Theory	**Exercises No. 6**	19.05.2016

[Tutor: Marc Sangel, Tutorial: Wednesday 25. May 2016, 16:00-17:30 in V2-200]

Exercise 6.1: Given the integration measure is defined as

$$\int \mathrm{d}\vec{v} \equiv \int_{-\infty}^{\infty}\left[\prod_i \frac{\mathrm{d}v_i}{\sqrt{2\pi}}\right]$$

derive the result for $e^{W(0)} = \int \mathrm{d}\vec{v}\, \exp\left[-\frac{1}{2}v_i A_{ij} v_j\right]$ from exercise 5.4. [answer: $e^{W(0)} = (\det A)^{-1/2}$].

Exercise 6.2: Consider the four-volume $V = L_0 L_1 L_2 L_3$ with periodic boundary conditions as in lecture.

(a) Show that it holds: $\lim_{V\to\infty} \frac{1}{V}\sum_P \tilde{f}(P) = \int \frac{\mathrm{d}^4P}{(2\pi)^4}\tilde{f}(P)$.

(b) We define a delta-function $\bar{\delta}(P)$ through the condition

$$\int_P \bar{\delta}(P-Q)\tilde{f}(P) = \tilde{f}(Q)\ ,$$

with $\int_P = \frac{1}{V}\sum_P$ or $\int_P = \int \frac{\mathrm{d}^4P}{(2\pi)^4}$. Write down $\bar{\delta}(P)$ for both cases.

(c) Show that the equation $\int_V \mathrm{d}^4x\, e^{iP\cdot x} = \bar{\delta}(P)$ is valid both for finite and infinite V.

Exercise 6.3: Show that the propagator

$$\Delta(x-y) = \int \frac{\mathrm{d}^4P}{(2\pi)^4}\frac{e^{iP\cdot(x-y)}}{P^2+m^2}$$

is finite for $x \neq y$. How does the function behave for $|x-y| \to \infty$? [Hint: choose the right basis for P and use complex analysis.]

Exercise 6.4: Show that $W[J] = \ln Z[J]$ generates the connected n-point Green functions for $n = 2, 4$ (use exercise 4.1).