Quantum Field Theory	**Exercises No. 7**	25.05.2016

[Tutor: Marc Sangel, Tutorial: Wednesday 01. June 2016, 16:00-17:30 in V2-200]

Exercise 7.1:

In lecture we have defined $Z[J]$, $W[J] = \ln Z[J]$, and $\Gamma[\varphi] = W[J] - \int d^4x\, \varphi(x) J(x)$, where $\varphi(x) = \frac{\delta W[J]}{\delta J(x)}$.

Starting from the Schwinger-Dyson equation for $Z[J]$,

$$0 = \left[-\mathcal{L}'_E \left(\frac{\delta}{\delta J(x)} \right) + J(x) \right] Z[J] \, ,$$

check that the following equations hold for $W[J]$ and $\Gamma[\varphi]$:

$$\text{(a)} \qquad \mathcal{L}'_E \left(\frac{\delta W[J]}{\delta J(x)} + \frac{\delta}{\delta J(x)} \right) = J(x) \, ,$$

$$\text{(b)} \qquad \mathcal{L}'_E \left(\varphi(x) + \int d^4y \, D[\varphi](x,y) \frac{\delta}{\delta \varphi(y)} \right) = -\frac{\delta \Gamma[\varphi]}{\delta \varphi(x)} \, ,$$

Here we denote $D[\varphi](x,y) \equiv \frac{\delta^2 W[J]}{\delta J(x) \delta J(y)}$ [use $\frac{\delta}{\delta J(x)} \mathbf{1} = 0$].

Exercise 7.2:

Consider $\mathcal{L}_E \equiv \frac{1}{2} \partial_\mu \phi \, \partial_\mu \phi + \frac{1}{2} m^2 \phi^2 + \frac{g}{3!} \phi^3$ from lecture with $\lambda = 0$. Write down the Schwinger-Dyson equations in a graphical way, up to including all $\mathcal{O}(g^2)$ terms. Indicate the order of each term in the expansion.