Quantum Field Theory **Exercises No. 8**

[Tutor: Marc Sangel, Tutorial: Wednesday 08. June 2016, 16:00-17:30 in V2-200]

Exercise 8.1:

Use the chain rule for functional integration

$$\frac{\delta}{\delta J(x)} = \int dz \frac{\delta\varphi(z)}{\delta J(x)} \frac{\delta}{\delta\varphi(z)}$$

to show that

$$\int dz G_E^{(2)}(x,z)\Gamma_E^{(2)}(z,y) = -\delta^{(4)}(x-y)$$

holds, relating the two-point Green's function and the two-point vertex function $\Gamma_E^{(2)}(z,y)$ defined in lecture. Also show that this translates into Fourier space as

$$\tilde{G}_E(p)\tilde{\Gamma}_E(p) = -1.$$

Exercise 8.2:

Many loop integrations can be simplified in the following so-called Feynman parametrisation. Please check that this identity holds:

$$\frac{1}{ab} = \int_0^1 \frac{\mathrm{d}t}{[at + b(1-t)]^2}\ , \quad a, b > 0\ .$$

Exercise 8.3:

Derive the relation for the superficial degree of divergence D for ϕ^r-theory in d dimensions from the lecture

$$D = d + \left(\frac{r}{2}(d-2) - d\right) N + \left(1 - \frac{d}{2}\right) E\ .$$

For the particular example with $r = 3$ and $d = 6$ write down all diagramms with $D \geq 0$ having maximally $N = 4$ vertices. Please also discuss all diagramms with $E = 4$ external legs and $N = 4$ vertices regarding their convergence, connectedness and if they are one-particle irreducible (1PI).

Exercise 8.4:

(a) Determine the following integral

$$I(m^2; d, A) \equiv \int \frac{\mathrm{d}^d P}{(2\pi)^d} \frac{1}{(P^2 + m^2)^A}$$

in dimensional regularisation [hint: use the Euler-Beta function].

(b) Please give $I(m^2; 4 - 2\epsilon, 1)$ for $\epsilon \ll 1$.