Quantum Field Theory	**Exercises No. 9**	09.06.2016

[Tutor: Marc Sangel, Tutorial: Wednesday 15. June 2016, 16:00-17:30 in V2-200]

Exercise 9.1:

Check the geometric series

$$\tilde{G}_E^{(2)}(p,-p) = G_0 \, \frac{1}{1+\Sigma G_0}$$

by inserting the diagrammatical expansion of $-\Sigma(p)$ from lecture up to including order $\mathcal{O}(\lambda^3)$ and compare it to that of $\tilde{G}_E^{(2)}(p,-p)$ from lecture.

Exercise 9.2:

Derive the order $\mathcal{O}(\lambda^2)$ for the 4-point function given in lecture. Can you obtain the combinatorics for the three terms contributing to the connected part?

Exercise 9.3:

The equation for the β-function in ϕ^4-theory is given to leading order by

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu}\lambda_R = \frac{3}{(4\pi)^2}\lambda_R^2 \, .$$

Verify that

$$\lambda_R(\mu) = \frac{(4\pi)^2}{3\ln(\mu_0/\mu)}$$

solves this equation, where μ_0 is a (dimensionful) constant.

Exercise 9.4:

Consider the integral

$$B(Q^2; m^2, m^2) \equiv \int \frac{\mathrm{d}^d P}{(2\pi)^d} \frac{1}{P^2+m^2} \frac{1}{(P+Q)^2+m^2} \, .$$

(a) Why does the result only depend on the square of the vector $Q^2 = \sum_\mu Q_\mu Q_\mu$, instead of all its components?

(b) Draw a Feynman diagram that would lead to such an integral.

(c) Use the Feynman-parametrisation from exercise 8.2 in order to compute $B(Q^2; m^2, m^2)$ as far as possible.