

[Tutor: Marc Sangel, Tutorial: Wednesday 22. June 2016, 16:00-17:30 in V2-200]

Exercise 10.1:

Let us define a Fock-space for Fermions in analogy to Bosons:

$$|\vec{k}_1 \dots \vec{k}_n; s_1 \dots s_n\rangle = \hat{a}_{\vec{k}_1}^{\dagger(s_1)} \dots \hat{a}_{\vec{k}_n}^{\dagger(s_n)} |0\rangle .$$

Please check the Pauli-principle:

$$|\vec{k}_1 \dots \vec{k}_i \dots \vec{k}_j \dots \vec{k}_n; s_1 \dots s_i \dots s_j \dots s_n\rangle = -|\vec{k}_1 \dots \vec{k}_j \dots \vec{k}_i \dots \vec{k}_n; s_1 \dots s_j \dots s_i \dots s_n\rangle .$$

Compute $\langle \vec{p}_1, \vec{p}_2; t_1, t_2 | \vec{k}_1, \vec{k}_2; s_1, s_2 \rangle$ and compare to the bosonic result from ex. 2.4.

Exercise 10.2:

Verify that the anti-commutation relations for creation and annihilation operators given in lecture imply the following equal time canonical anti-commutation relations

$$\{\hat{\psi}_\alpha(x^0, \vec{x}), \hat{\Pi}_\beta(x^0, \vec{y})\} = i\delta^{(3)}(\vec{x} - \vec{y})\delta_{\alpha\beta} , \quad \alpha, \beta = 1, \dots, 4 ,$$

where $\hat{\Pi}_\beta \equiv i\hat{\psi}_\beta^\dagger = i\hat{\psi}_\alpha \gamma_{\alpha\beta}^0$ [Hint: use the completeness relation for the spinors $u_\alpha(\vec{p}, s)$, $v_\beta(\vec{p}, s)$, and substitute $\vec{p} \rightarrow -\vec{p}$ at some point].

Exercise 10.3:

Let M be a 2×2 matrix with bosonic entries, and define the following two vectors $c^{*T} \equiv (c_1^*, c_2^*)$, $c^T \equiv (c_1, c_2)$ with fermionic Grassmann variables. Compute the following Grassmann integral with the bilinear "action" $S_E = c^{*T} M c$:

$$I(M) \equiv \int dc_1^* dc_1 dc_2^* dc_2 \exp(-S_E) .$$

Compare the answer to ex 6.1 - what happens when you consider Gaussian integrals over complex bosonic variables there?

Exercise 10.4:

Compute the following averages with S_E defined in the previous ex. 10.3:

- (a) $\langle c_1 c_1^* \rangle \equiv \frac{\int dc_1^* dc_1 dc_2^* dc_2 c_1 c_1^* \exp(-S_E)}{\int dc_1^* dc_1 dc_2^* dc_2 \exp(-S_E)} ,$
- (b) $\langle c_1 c_2^* \rangle ,$
- (c) $\langle c_2 c_1^* \rangle ,$
- (d) $\langle c_2 c_2^* \rangle .$

Can you find a compact way of generating these averages?