Quantum Field Theory	**Exercises No. 11**	23.06.2016

[Tutor: Marc Sangel, Tutorial: Wednesday 29. June 2016, 16:00-17:30 in V2-200]

Exercise 11.1:

We define expectation values as

$$\langle ... \rangle_0 \equiv \frac{\int \{\prod_i \mathrm{d}c_i^* \mathrm{d}c_i\} (...) \exp(-\sum_{p,q} c_p^* A_{pq} c_q)}{\int \{\prod_i \mathrm{d}c_i^* \mathrm{d}c_i\} \exp(-\sum_{p,q} c_p^* A_{pq} c_q)} \ ,$$

with c_j^*, c_k being Grassmann variables.

What do you get for $\langle c_k c_l c_m c_n \rangle_0,\ \langle c_k c_l c_m c_n^* \rangle_0$, and $\langle c_k c_l c_m^* c_n^* \rangle_0$?

Exercise 11.2:

Consider the Euclidean Dirac-gamma matrices from lecture. Show the following properties:

a) $\mathrm{Tr}\,[\gamma_5] = 0$

b) the trace of an odd number of γ's vanishes (hence $\mathrm{Tr}\,[\gamma_5 \gamma_\mu] = 0 = \mathrm{Tr}\,[\gamma_5 \gamma_\mu \gamma_\nu \gamma_\rho]$

c) $\mathrm{Tr}\,[\gamma_5 \gamma_\mu \gamma_\nu] = 0$

d) $\mathrm{Tr}\,[\gamma_5 \gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma] = N \varepsilon_{\mu\nu\rho\sigma}$ with $\mathrm{Tr}\,[\gamma_0^2] = N$. Here $\varepsilon_{\mu\nu\rho\sigma}$ is the totally antisymmetric epsilon tensor with $\varepsilon_{1234} = +1$.

Exercise 11.3:

Verify the Schwinger propagator for the Dirac field

$$\langle 0 | T\{\hat{\psi}_\alpha(x) \hat{\bar{\psi}}_\beta(y)\} | 0 \rangle = \int \frac{d^4 P}{(2\pi)^4} e^{iP\cdot(x-y)} \frac{[-i\gamma_\mu^E P_\mu + m\mathbf{1}]_{\alpha\beta}}{P^2 + m^2} \ .$$

Exercise 11.4:

Show that neither

$$\mathcal{L} = (\partial_\mu \Phi^*)(\partial^\mu \Phi) - m^2 \Phi^* \Phi \quad \text{nor} \quad \mathcal{L} + g J_\mu A^\mu$$

with $J_\mu = i\Phi^*(\partial_\mu \Phi) - i(\partial_\mu \Phi^*)\Phi$ from lecture are invariant under the following local gauge transformations of the complex scalar field $\Phi(x)$ and real vector field $A^\mu(x)$:

$$\Phi(x) \to \Phi'(x) = e^{iw(x)} \Phi(x) \ , \quad A^\mu(x) \to A^{\mu\prime}(x) - A^\mu(x) + \partial^\mu w(x)/g$$