Quantum Field Theory	**Exercises No. 12**	29.06.2016

[Tutor: Marc Sangel, Tutorial: Wednesday 06. Juli 2016, 16:00-17:30 in V2-200]

Exercise 12.1:

Show that the following representation holds

$$f^{abc} = -2i\mathrm{Tr}([T^a, T^b]T^c) \; ,$$

where T^a are the generators of $SU(N)$ and f^{abc} its structure constants. Use this to show that the structure constants are totally anti-symmetric in their indices.

Exercise 12.2:

Given pure Yang-Mills theory with the Lagrange density

$$\mathcal{L}_{YM} \equiv -\frac{1}{4} F^{a\mu\nu} F^a_{\mu\nu} \; ,$$

with the components of the field strength tensor $F^a_{\mu\nu}$ defined as in lecture. What are the classical equations of motion for the gauge field $A_\mu(x)$?

Exercise 12.3:

Show that through integration by parts the following Lagrangian of Maxwell's theory with a parameter dependent gauge fixing can be rewritten as follows:

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{2\alpha} (\partial_\mu A^\mu)^2 = \frac{1}{2} A_\mu \left[g_{\mu\nu} \partial_\rho \partial^\rho - (1 - \frac{1}{\alpha}) \partial_\mu \partial_\nu \right] A^\nu .$$

Derive this differential operator in momentum space $-\tilde{\Delta}^{-1}_{\mu\nu}$ and find its inverse $D_{\mu\nu}$ such that the following holds:

$$-D^{\mu\nu} \tilde{\Delta}^{-1}_{\nu\rho} = \delta^\mu_\rho .$$

Exercise 12.4:

Let us consider $C(x) \equiv \mathrm{Tr}[F_{\mu\nu}(x) F_{\rho\sigma}(x)] \epsilon^{\mu\nu\rho\sigma}$, where $\epsilon^{\mu\nu\rho\sigma}$ is the totally antisymmetric 4-dimensional Levi-Civita tensor (=+1, if $\mu\nu\rho\sigma$ is an even permutation of 0123, =-1 if it is an odd permutation, and =0 otherwise).

(a) Show that $C(x)$ is gauge invariant.

(b) Demonstrate that $C(x)$ is Lorentz invariante for proper Lorentz transformations Λ with $\det \Lambda = +1$.

(c) Verify that $C(x)$ satisfies

$$\begin{aligned} C(x) &= \partial_\mu K^\mu(x) \; , \\ K^\mu(x) &= 2\epsilon^{\mu\nu\rho\sigma} \Big(A^a_\nu \partial_\rho A^a_\sigma + \frac{g}{3} f^{abc} A^a_\nu A^b_\rho A^c_\sigma \Big) \; . \end{aligned}$$

Hence it is a surface (or topological) term after integration over $\int \mathrm{d}^4 x$.