| Quantum Field Theory | **Exercises No. 14** | 14.07.2016 |
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[ Tutor: Marc Sangel, Tutorial: Wednesday 20. Juli 2016, 16:00-17:30 in V2-200 ]

**Exercise 14.1:** The proper vertex function $\Gamma_\mu(p,q,r)$ is defined by the following Fourier transform:

$$-ig(2\pi)^4\delta^{(4)}(r-p-q)\Gamma_\mu(p,q,r) \equiv \int d^4z d^4x d^4y e^{i(rx-py-qz)} \frac{\delta^3\Gamma[\psi\,\bar\psi, A_\nu]}{\delta\bar\psi(x)\delta\psi(y)\delta A_\mu(z)}\Big|_{\psi=\bar\psi=A=0}$$

with $\Gamma[\psi\,\bar\psi, A_\nu]$ defined in lecture.

1. Complete the derivation of the Ward-Takahashi identity for the proper vertex function starting from the identity for $\Gamma[\psi\,\bar\psi, A_\nu]$ from lecture;

$$-\frac{1}{\alpha}\Box\,\partial^\mu A_\mu + \partial_\mu\frac{\delta\Gamma}{\delta A_\mu} + ig\left(\frac{\delta\Gamma}{\delta\psi}\psi - \frac{\delta\Gamma}{\delta\bar\psi}\bar\psi\right) = 0$$

2. Check that the Ward identity is satisfied to leading order: express the integrand above through the generating functional $Z[\eta,\bar\eta,J_\nu]$ and the leading order photon and electron propagators.

**Exercise 14.2:** Draw all possible divergent 1-loop diagrams in QED and give their superficial degree of divergence. What does $D=0$ mean?

**Exercise 14.3:** Generalize the steps in ex. 8.4 to compute the following integral in Minkowski space ($p^2 = p_0^2 - p_j p_j$):

$$\int \frac{d^dp}{(p^2+2p\cdot q-m^2)^A} = i(-\pi)^{d/2}\frac{\Gamma\left(A-\frac{d}{2}\right)}{\Gamma(A)[-q^2-m^2]^{A-d/2}}\ .$$

Use this result to compute the following two integrals:

$$I_1 = \int d^dp\frac{p_\mu}{(p^2+2p\cdot q-m^2)^A}\quad,\quad I_2 = \int d^dp\frac{p_\mu p_\nu}{(p^2+2p\cdot q-m^2)^A}\ .$$

**Exercise 11.4:** Compute the photon self-energy also called vacuum polarization $\Pi_{\mu\nu}(k)$ in Lorentz gauge $\alpha=1$ using dimensional regularization. Use the Feynman parameterization ex. 8.2 as well as the above ex. 14.3.