Universität Bielefeld

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Symmetries in Physics – Exercise Sheet 4

Exercise 4.1

An example in many-body Quantum Mechanics: We consider a quantum system with n identical particles. The symmetrization is performed via the permutation group S_n.

(i) Let us assume that the many-particle quantum state can be approximated by single particle states $\{|\psi_j\rangle\}_{j=1,\ldots,n}$. What is the explicit form of the many-particle quantum state $|\psi_1,\ldots,\psi_n\rangle$?

(ii) Let us assume that the many-particle quantum state can be approximated by two-particle states $\{|\psi_i,\psi_j\rangle\}_{1\leq i<j\leq n}$ with $n\in 2\mathbb{N}$ (this happens when you have Cooper-pairs). What is the explicit form of the many-particle quantum state $|\psi_1,\ldots,\psi_n\rangle$? Interpret the coset structure S_n/S_2 as well as the conjugacy classes of S_n in the resulting expression for $|\psi_1,\ldots,\psi_n\rangle$.

(iii) Let us assume that the many-particle quantum state can be approximated by m-particle states $\{|\psi_{i_1},\ldots,\psi_{i_m}\rangle\}_{1\leq i_1<\ldots<i_m\leq n}$ with $n\in m\mathbb{N}$ and $m\in\mathbb{N}$. What is the explicit form of the many-particle quantum state $|\psi_1,\ldots,\psi_n\rangle$? Interpret the coset structure S_n/S_m as well as the conjugacy classes of S_n in the resulting expression for $|\psi_1,\ldots,\psi_n\rangle$.

(iv) What happens if we have a mixed situation of a combination of different sized particle states? Explain your answer with help of the conjugacy classes of S_n.

(v) What happens if we have bosons instead of fermions?

Exercise 4.2

An application in Electrostatics: Let us consider two ideal conducting half-planes. One half-plane, E_1, should be the x-z-half-plane with $x>0$ and z arbitrary such that its normal vector is given by $\mathbf{n}_1^T=(0,1,0)$ and its edge is the z-axis. The other half-plane, E_2, has a normal vector $\mathbf{n}_2^T=(-\sin\varphi,\cos\varphi,0)$ and its edge is also the z-axis. Between the two planes is an electrically charged particle (assume an electron). The aim is to derive the electrostatic potential $V(\mathbf{r})$ of this system.

(i) Let R_1 be the reflection at the half-plane E_1 and R_2 the reflection at E_2. Explain why at both reflections R_1 and R_2 the effective electrical charge density (real charges plus virtual charges) is antisymmetric, i.e. $\rho(\mathbf{r})=-\rho(R_j\mathbf{r})$.

(ii) Let us assume $\varphi=\pi/n$ with $n\in\mathbb{N}$. Explain why the dihedral group D_n is a symmetry group of the system. What is its group action on the system? Moreover, construct with the knowledge gained the electrostatic potential $V(\mathbf{r})$ for the case $\varphi=\pi/n$.

(iii) Let us assume $\varphi=m\pi/n$ with $m,n\in\mathbb{N}$ and n is no divisor of m. Explain why the circular groups C_{mn} and C_n are symmetry groups of the system. What is their group action on the system, i.e. how are they embedded? Notice, that C_n can be embedded in at least two different ways! Furthermore, calculate the coset C_{mn}/C_n and show that C_n is a normal subgroup of C_m. Interpret the action of the corresponding quotient group on the system. Moreover, construct with the knowledge gained the electrostatic potential $V(\mathbf{r})$ for the case $\varphi=m\pi/n$.