



**UNIVERSITÄT
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Faculty of Physics



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Seminar

Bielefeld - Melbourne Random Matrices

Meng Yang
University of Copenhagen

Strong Asymptotics of Planar Orthogonal Polynomials:
Gaussian Weight Perturbed by Point Charges

We consider the planar orthogonal polynomials $\{p_n(z)\}$ with respect to the measure supported on the complex plane

$$e^{-N|z|^2} \prod_{j=1}^{\nu} |z - a_j|^{2c_j} dA(z)$$

where dA is the Lebesgue measure of the plane, N is a positive constant, $\{c_1, \dots, c_\nu\}$ are nonzero real numbers greater than -1 and $\{a_1, \dots, a_\nu\} \subset \mathbb{D} \setminus \{0\}$ are distinct points inside the unit disk. The orthogonal polynomials are related to the interacting Coulomb particles with charge $+1$ for each, in the presence of extra particles with charge $+c_j$ at a_j . For fixed c_j , these can be considered as small perturbations of the Gaussian weight. When $\nu = 1$, in the scaling limit $n/N = 1$ and $n \rightarrow \infty$, we obtain strong asymptotics of $p_n(z)$ via a matrix Riemann-Hilbert problem. From the asymptotic behavior of $p_n(z)$, we find that, as we vary c_1 , the limiting distribution of zeros behaves discontinuously at $c_1 = 0$. We observe that the generalized Szegő curve (a kind of potential theoretic skeleton) also behaves discontinuously at $c_1 = 0$. We also derive the strong asymptotics of $p_n(z)$ for the case of $\nu > 1$ by applying the nonlinear steepest descent method on the matrix Riemann-Hilbert problem of size $(\nu + 1) \times (\nu + 1)$. This talk is based on joint work with Seung-Yeop Lee.

Wednesday, 25 November 2020, 0900 hrs CET

ZOOM Conference Call – Please contact Thorsten Neuschel
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